PG – 252

I Semester M.Sc. Degree Examination, January/February 2018
(CBCS Scheme)
MATHEMATICS
M103T : Topology – I

Time : 3 Hours   Max. Marks : 70

Instructions : *i) Answer **any five** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Let X be an infinite set and $x_0 \in X$, then prove that $X - \{x_0\}$ is infinite.

   b) Define countable set. Prove that a set X is infinite if and only if either $X = \phi$ or X is in one-to-one correspondence with some $¥_k$, where $¥_k = \{1, 2, 3, \ldots, k\}$ set of all natural numbers from 1 to k. **(6+8)**

2. a) Let X and Y be sets. If X is equivalent to a subset of Y and Y is equivalent to a subset of X, then prove that X and Y are equivalent.

   b) If P(A) denote the power set of a set A, then prove that card $(P(A)) = 2^{card(A)}$. **(9+5)**

3. a) Define a metric space. If d is a metric on X, prove that $\rho(x, y) = \frac{d(x, y)}{1 + d(x, y)}$, $\forall x, y \in X$ is a metric on X.

   b) Prove that a subspace Y of complete metric space (X, d) is complete if and only if it is closed. **(7+7)**

4. a) State and prove contraction mapping theorem.

   b) State and prove Cantor's intersection theorem. **(6+8)**

5. a) Prove that an isometry is a homeomorphism but not conversely.

   b) Prove that every metric space has a completion. **(7+7)**

6. a) Prove that every metric space is a topological space.

   b) Prove that a set is open if and only if it is neighbourhood of each of its points.

   c) Prove that a point $x \in (X, \mathcal{T})$ belongs to the closure of a set A if and only if every open set G which contains x has a non-empty intersection with A. **(4+4+6)**

P.T.O.

7. a) Prove that $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is continuous at $x \in X$ if and only if V is a neighbourhood of $f(x) \Rightarrow f^{-1}(V)$ is a neighbourhood of x.

   b) Prove that a bijective function $f : (X, \mathcal{T}) \to (Y, \mathcal{U})$ is a homeomorphism if and only if $f(A^0) = [f(A)]^0, \forall A \subset X$. (6+8)

8. a) Prove that a topological space $(X, \mathcal{T})$ is connected if and only if the only continuous map from X to the 2-point space over the constant map..

   b) Prove that the components of a totally disconnected space are its points. (7+7)